APPLICATIONS : FUNCTIONS OF MULTIPLE VARIABLES

Question 1. Your automobile assembly plant has a **Cobb-Douglas production function** given by

$$q = 100x^{0.3}y^{0.7}$$

where q is the number of automobiles it produces per year, x is the number of employees, and y is the monthly assemblyline budget (in thousands of dollars).

Annual operating costs amount to an average of \$60 thousand per employee plus the operating budget of \$12y thousand. Your annual budget is \$1, 200, 000. How many employees should you hire and what should your assembly-line budget be to maximize productivity? What is the productivity at these levels?

Question 2. American Airlines American Airlines requires that the total outside dimensions (length + width + height) of a checked bag not exceed 62 inches What are the dimensions of the largest volume bag that you can check on an American flight?

Question 3. The only grocery store in a small rural community carries two brands of orange juice, a local brand that it obtains at the cost of 30 cents per can and a well-known national brand that it obtains at the cost of 40 cents per can. The grocer estimates that if the local brand is sold for x cents per can and the national brand for y cents per can, approximately $70 - 5x + 4y$ cans of the local brand and $80 + 6x - 7y$ cans of the national brand will be sold each day. How should the grocer price each brand to maximize the profit from the sale of the juice?

The cost function (in cents) $C(x, y)$ is given by

$$C(x, y) = 30(70 - 5x + 4y) + 40(80 + 6x - 7y)$$

and the revenue function (in cents) $R(x, y)$ is given by

$$R(x, y) = x(70 - 5x + 4y) + y(80 + 6x - 7y)$$

The the profit functions is given by

$$\begin{aligned} P(x, y) = R(x, y) - C(x, y) &= (x - 30)(70 - 5x + 4y) + (y - 40)(80 + 6x - 7y) \\ &= -5x^2 + 10xy - 20x - 7y^2 + 240y - 5300 \end{aligned}$$

Then,

$$P_x = -10x + 10y - 20; \quad P_y = 10x - 14y + 240; \quad P_{xx} = -10x; \quad P_{yy} = -14; \quad P_{xy} = 10$$

The critical points for the function P given above is obtained by simulataneously solving the equations $P_x = 0$ and $P_y = 0$.

$$\begin{aligned} P_x = 0 &\implies -10x + 10y - 20 = 0 \implies -x + y = 2 \\ P_y = 0 &\implies 10x - 14y + 240 = 0 \implies 5x - 7y = -120 \end{aligned}$$

Then solve these equations simultaneously, and the function $P(x, y)$ has a critical point at

$$x = 53; \quad y = 55$$

To determine whether P has a maximum or minimum at this point, we need to use the second derivative test:

$$H(53, 55) = P_{xx}(53, 55) \cdot P_{yy}(53, 55) - [P_{xy}(53, 55)]^2 = -10 \cdot (-14) - 10^2 = 40 > 0$$

and since $P_{xx}(53, 55) = -10 < 0$, it follows that $P(x, y)$ has a relative maximum when $(x, y) = (53, 55)$.

Therefore the grocer can maximize profit by selling the local brand for 53 cents per can and the national brand for 55 cents per can.

Question 4. To increase business at OHaganBooks.com, you have purchased banner ads at well-known Internet portals and have advertised on television. The following interaction model shows the average number h of hits per day as a function of monthly expenditures x on banner ads and y on television advertising (x and y are in dollars).

$$h(x, y) = 1,800 + 0.05x + 0.08y + 0.00003xy$$

a) Based on your model, how much traffic can you anticipate if you spend $\$2,000$ per month for banner ads and $\$3,000$ per month on television advertising?

$$h(2000, 3000) = 1800 + 0.05(2000) + 0.08(3000) + 0.00003(2000)(3000) = 2320 \text{ hits per day}$$

b) Evaluate $\frac{\partial h}{\partial y}$, specify its units of measurement, and indicate whether it increases or decreases with increasing x.

$$\frac{\partial h}{\partial y} = h_y = 0.08 + 0.00003x \text{ hits per dollar spent on television advertising per month}$$

This increases as x increase.

c) How much should the company spend on banner ads to obtain 1 hit per day for each $\$5$ spent per month on television advertising?

From part b) we see that the question says

$$h_y = \frac{1}{5}.$$

Therefore from part b),

$$0.2 = 0.08 + 0.0003x \implies x = \$4000 \text{ per month}$$